Kuta Software - Infinite Calculus	Name___________________________________

Area Under a Curve Using Limits of Sums

Date________________ Period____

Evaluate each sum.

1) $\sum_{k=1}^{n} 18k$

2) $\sum_{k=1}^{n} (2k+5)$

3) $\sum_{k=1}^{n} 8k^2$

4) $\sum_{k=1}^{n} (k^2+4)$

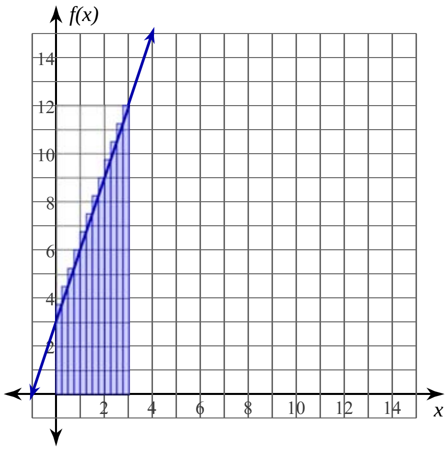
Evaluate each limit.

5) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n} + \frac{2k}{n^2}\right)$

6) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{5}{n} + \frac{k^3}{n^4}\right)$

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as *n* goes to ∞ of the upper sum.

7) $f(x) = 3x + 3$; $[0, 3]$



8) $f(x) = x^2 + 5$; $[1, 3]$

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Area Under a Curve Using Limits of Sums

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Evaluate each sum.

1) $\sum_{k=1}^{n} 18k$

$18 \cdot \sum_{k=1}^{n} k$

$18 \cdot \frac{n(n+1)}{2}$

$9n^2 + 9n$

2) $\sum_{k=1}^{n} (2k+5)$

$2 \cdot \sum_{k=1}^{n} k + 5 \cdot \sum_{k=1}^{n} 1$

$2 \cdot \frac{n(n+1)}{2} + 5n$

$n^2 + 6n$

3) $\sum_{k=1}^{n} 8k^2$

$8 \cdot \sum_{k=1}^{n} k^2$

$8 \cdot \frac{n(n+1)(2n+1)}{6}$

$\frac{8n^3}{3} + 4n^2 + \frac{4n}{3}$

4) $\sum_{k=1}^{n} (k^2+4)$

$\sum_{k=1}^{n} k^2 + 4 \cdot \sum_{k=1}^{n} 1$

$\frac{n(n+1)(2n+1)}{6} + 4n$

$\frac{n^3}{3} + \frac{n^2}{2} + \frac{25n}{6}$

Evaluate each limit.

5) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n} + \frac{2k}{n^2}\right)$

$\lim_{n\to\infty} \left(\frac{2}{n} \cdot \sum_{k=1}^{n} 1 + \frac{2}{n^2} \cdot \sum_{k=1}^{n} k\right)$

$\lim_{n\to\infty} \left(\frac{2}{n} \cdot n + \frac{2}{n^2} \cdot \frac{n(n+1)}{2}\right)$

$\lim_{n\to\infty} \left(3 + \frac{1}{n}\right)$

3

6) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{5}{n} + \frac{k^3}{n^4}\right)$

$\lim_{n\to\infty} \left(\frac{5}{n} \cdot \sum_{k=1}^{n} 1 + \frac{1}{n^4} \cdot \sum_{k=1}^{n} k^3\right)$

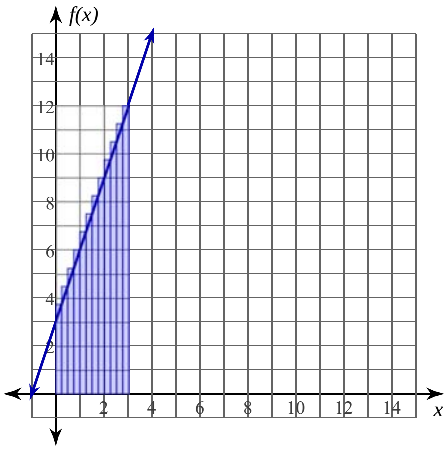
$\lim_{n\to\infty} \left(\frac{5}{n} \cdot n + \frac{1}{n^4} \cdot \frac{n^2 \cdot (n+1)^2}{4}\right)$

$\lim_{n\to\infty} \left(\frac{21}{4} + \frac{1}{2n} + \frac{1}{4n^2}\right)$

$\frac{21}{4} = 5.25$

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as *n* goes to ∞ of the upper sum.

7) $f(x) = 3x + 3;\ [0, 3]$



$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(3\left(0 + \frac{3-0}{n}\cdot k\right) + 3\right)\cdot\frac{3-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{9}{n} + \frac{27k}{n^2}\right)$$

$$\lim_{n\to\infty} \left(\frac{9}{n}\cdot\sum_{k=1}^{n} 1 + \frac{27}{n^2}\cdot\sum_{k=1}^{n} k\right)$$

$$\lim_{n\to\infty} \left(\frac{9}{n}\cdot n + \frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$\lim_{n\to\infty} \left(\frac{45}{2} + \frac{27}{2n}\right)$$

$$\frac{45}{2} = 22.5$$

8) $f(x) = x^2 + 5;\ [1, 3]$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(1 + \frac{3-1}{n}\cdot k\right)^2 + 5\right)\cdot\frac{3-1}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{12}{n} + \frac{8k}{n^2} + \frac{8k^2}{n^3}\right)$$

$$\lim_{n\to\infty} \left(\frac{12}{n}\cdot\sum_{k=1}^{n} 1 + \frac{8}{n^2}\cdot\sum_{k=1}^{n} k + \frac{8}{n^3}\cdot\sum_{k=1}^{n} k^2\right)$$

$$\lim_{n\to\infty} \left(\frac{12}{n}\cdot n + \frac{8}{n^2}\cdot\frac{n(n+1)}{2} + \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$\lim_{n\to\infty} \left(\frac{56}{3} + \frac{8}{n} + \frac{4}{3n^2}\right)$$

$$\frac{56}{3} \approx 18.667$$